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ASSESMENT OF ULTIMATE LOAD IN PLASTIC ANALYSIS BY USING THE BOND-GRAPH METHOD

BY

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Abstract. The present paper proves the bond-graph method large range of application in resolving not only dynamic, but also static problems. It is shown the manner of applying this method in the plastic analysis of structural elements made of ductile materials, for determining the ultimate load, which produces the system collapse.

Key words: bond-graph; plastic analysis; plastic hinge; ultimate load.

1. General Considerations

The structural elements made of ductile materials, like mild steel, have a very important carrying capacity reserve in the elastoplastic range, conferred by the yielding plateau and the hardening zone. During material evolution in this range, there are important redistributions of stresses over the section of the element and redistributions of internal forces between the sections of the element. The final stage is reached when the system becomes a mechanism with one degree of freedom. The load which produces the collapse mechanism, called ultimate load, can be determined not only with the methods provided by Mechanics of Materials,

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but also with the bond-graph method, as we show in the paper. The bond-graph model will be adopted for a statically indeterminate steel beam.

2. Plastic Analysis of Steel Beams

As well-known, the plastic analysis is based on Prandtl's diagram. According to this diagram, which describes the behaviour of an elastic-perfectly plastic material, the maximum normal stress which develops in the material is the yield stress, σ_y.

When the most loaded section of a steel beam becomes a fully plastic one (all stresses reach the yield value), a plastic hinge occurs at that section.

Unlike common hinges, where the bending moment is zero, in a plastic hinge, the bending moment equals the so called fully plastic moment:

$$M_{pl} = \sigma_y \cdot W_{pl} \tag{1}$$

where W_{pl} is the plastic modulus of section.

The beam loses a constraint, which in case of statically determinate beams it is the moment when the element becomes a mechanism with one degree of freedom and the ultimate stage is attained.

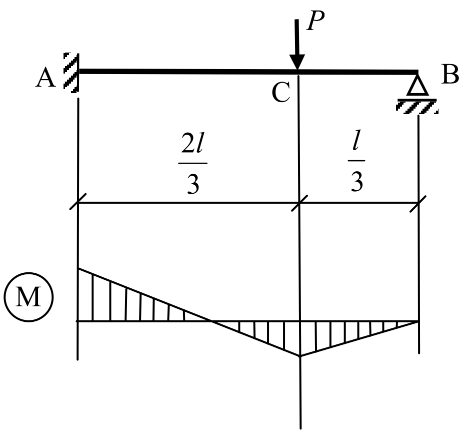

Fig. 1 − The statically indeterminate beam and its bending moment diagram.

In case of statically indeterminate beams to the "n^{th}" degree, (n+1) plastic hinges transform the beam into a mechanism with one degree of freedom.

Based on this remark, in case of a statically indeterminate beam to the "n^{th}" degree, the ultimate load, P_u, could be defined as the minimum load which leads to (n+1) plastic hinges along the beam (Ibănescu & Toma, 2013).

Let us consider the statically indeterminate beam to the first degree shown in Fig. 1.

The most loaded sections are A and C, according to the bending moment diagram (Fig. 1).

The beam becomes a mechanism with one degree of freedom when plastic hinges occur at the two sections, previously mentioned.

The ultimate load assessment is based on the collapse mechanism shown in Fig. 2.

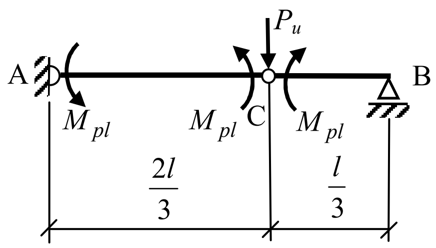

Fig. 2 − The model of the beam corresponding to the collapse moment.

3. The Bond-Graph Model

The bond-graph method is used in the analysis of dynamic systems and is based on system power balance (Borutsky, 2010; Karnopp *et al.*, 2006; Păstrăvanu & Ibănescu, 2001). In order to apply this method in the plastic analysis of statically indeterminate beams, we have to presume that under ultimate load action, the beam becomes a mechanism made of two pinned bars (1 and 2), as pictured in Fig. 3 (Ibănescu & Ibănescu, 2004; Ibănescu & Ungureanu, 2014).

The bar no. 1 is presumed to rotate around point A and its moment of inertia with respect to this point is J_1. The hypothetic rotation of bar no. 1 around point A is defined by the angular velocity, ω_1. Two moments, equal to the fully plastic moment, M_{pl}, apply upon it. The bar no. 2 rotates around point B, the angular velocity being ω_2 and the moment of inertia J_2 . A single moment, equal to the fully plastic moment, M_{pl}, acts upon it, as previously presented. Under the action of load P_u, point C has a hypothetic velocity, v. Between these velocities, the following relations exist:

$$\omega_1 = \frac{v}{2l/3} \tag{2}$$

and

$$\omega_2 = \frac{v}{l/3} \tag{3}$$

In the bond-graph model represented in Fig. 4, a 1-junction is considered for each of the following velocities: ω_1, ω_2 and v. Two effort sources, corresponding to the fully plastic moments, M_{pl}, which act upon bar no. 1, are connected to the 1-junction corresponding to angular velocity, ω_1. An inertial element, which corresponds to J_1, is connected too. At the 1-junction adopted for velocity v, an effort source for the ultimate load, P_u, is connected. An effort source, for the fully plastic moment, M_{pl}, which acts upon bar no. 2, is considered for the 1-junction corresponding to the angular velocity ω_2.

Between the 1-junctions corresponding to velocities ω_1, ω_2 and the 1-junction corresponding to velocity v, transformers are introduced. The parameters of the two transformers are $2l/3$ and $l/3$, respectively, according to (2) and (3).

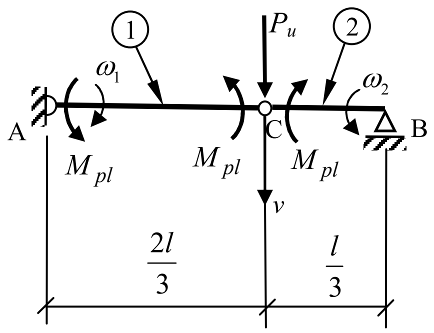

Fig. 3 − The model of the beam in the ultimate stage as a dynamic mechanism.

The power is provided by force P_u and it is distributed to the inertial elements, as shown in Fig. 4. As the three plastic moments, M_{pl}, and the bar angular velocities, have opposite directions, they are modeled as sinks.

After the causal assignment of the bond sources, an integral causality is assigned to element I, defined by parameter J_1. Therefore, all the bonds gain the causal stroke and the element I, defined by parameter J_2, results in derivative causality (Borutsky, 2010; Ibănescu, 2011).

As we have a single energy storage element in integral causality, a single differential equation for system dynamics results:

$$\dot{p}_6 = e_3 - e_4 - e_5 = e_3 - 2M_{pl} \tag{4}$$

The effort constitutive equation of the transformer defined by parameter $2l/3$, is:

$$e_3 = \frac{2l}{3} \cdot e_2 \tag{5}$$

The effort equilibrium equation, for the 1-junction corresponding to velocity *v*, leads to the relation:

$$e_2 = e_1 - e_7 = P_u - e_7 \tag{6}$$

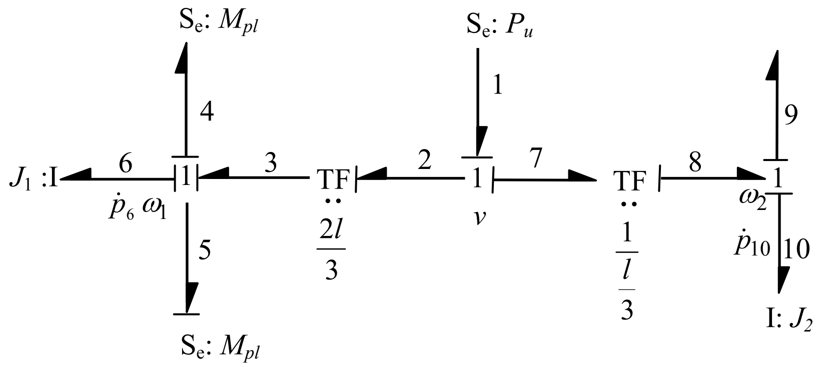

Fig. 4 − The bond graph model of the system.

By substituting (5) and (6) in (4), it results:

$$\dot{p}_6 = \frac{2l}{3} \cdot P_u - \frac{2l}{3} \cdot e_7 - 2 \cdot M_{pl} \tag{7}$$

The effort constitutive equation of the transformer defined by parameter 1/(*l*/3) is:

$$e_7 = \frac{1}{\frac{l}{3}} \cdot e_8 \tag{8}$$

The effort equilibrium equation of 1-junction, which corresponds to the angular velocity ω_2, leads to:

$$e_8 = e_9 + e_{10} = M_{pl} + \dot{p}_{10} \tag{9}$$

By substituting (8) and (9) in (7), the following differential equation is determined:

$$\dot{p}_6 = \frac{2l}{3} \cdot P_u - 4 \cdot M_{pl} - 2 \cdot \dot{p}_{10} \tag{10}$$

As the system is in equilibrium, the terms $\dot{p}_6$ and $\dot{p}_{10}$, corresponding to the inertia of the moving elements, are equal to zero. In these circumstances, eq. (10) becomes:

$$\frac{2l}{3} \cdot P_u - 4 \cdot M_{pl} = 0 \tag{11}$$

From this equation, the ultimate load, P_u, is:

$$P_u = \frac{6M_{pl}}{l}, \tag{12}$$

the same result, as that one obtained by using Mechanics of Material procedures.

4. Conclusions

1. The work proves that in certain circumstances, the bond-graph method can be applied for resolving not only dynamic problems, but also static ones.
2. The method can be used for a very large category of problems, including classical problems in Mechanics of Materials, like the plastic analysis of beams.

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CALCULUL FORŢEI ULTIME ÎN DOMENIUL PLASTIC
FOLOSIND METODA BOND-GRAPH

(Rezumat)

Metoda bond-graph este utilizată în mod curent la rezolvarea problemelor de dinamica sistemelor. Autorii au reuşit extinderea aplicării metodei la rezolvarea unor probleme de statică. Folosind ideile şi rezultatele cercetărilor anterioare pe această temă, prezenta lucrare extinde folosirea modelului bond-graph în calculul forţei ultime în domeniul plastic pentru o grindă metalică static nedeterminată, problemă clasică din Rezistenţa materialelor.